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THE INFLUENCE OF LATERAL JETS, SIMPLE OR COMBINED WITH LONGITUDINAL JETS, UPON THE WING LIFTING CHARACTERISTICS

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INTRODUCTION

The problem of longitudinal jet flaps has constituted the object of several theoretical and experimental studies presented partially at the Second International Congress of Aeronautical Sciences[1]; in an equally comprehensive form and including many other aspects, this problem is again discussed in a new communication[2] at this third Congress.

From these papers, as well as from the investigations carried out at the Institute of Applied Mechanics in Bucharest,[3] one may notice that for wings of low aspect ratio, the lift is substantially smaller. This effect becomes more marked for the wings of very low aspect ratio, as is for instance the case of the wings employed in supersonic aviation. In order to overcome this disadvantage so marked for wings of moderate, low, or very low aspect ratios, we have treated some new jet flap aspects, applied to this class of wings. Indeed, the present paper includes a new element which to the author's knowledge has not been so far considered. This new element is the effect of the lateral fluid jets, horizontal or tilted with respect to the horizontal. These jets, simple or combined with longitudinal jets, have a quite remarkable influence upon the aerodynamic characteristics of the wings of low aspect ratio, but mainly on the wings of very small aspect ratio. We shall refer particularly to the lift, which will be considered as a basic parameter for STOL aircrafts.

We shall consider first the action of the lateral jet on a rectangular wing.

THEORETICAL CONSIDERATIONS ON LATERAL JETS

The effect of lateral jets and their influence in combination with longitudinal jets constitute the object of the present paper. Therefore we shall try to present several theoretical considerations on the simple case of lateral jets which constitutes the basic concept of this problem and whose scheme may be easily situated within the theory of the wings of finite span. In this sense we shall refer to our previous paper[4] and consider a rectangular wing of span b and chord c, having hence the aspect ratio

$$\lambda_0 = \frac{b}{c} \tag{1}$$

which will be considered small or moderate.

Let us further consider a fluid jet of the same chord as the wing, having the same incidence (Fig. 1). In this case, the fluid jet behaves like a flat plate, on the surface of which the corresponding pressure develops. There results a lift coefficient C_l, which for the sake of simplicity will be assumed to have approximately a triangular variation, with the vertex at the leading edge. Thus, assuming that at point y on the span, the mean unit lift is C_l, corresponding to the circulation Γ, that is

$$\rho_\infty U_\infty \Gamma dy = \frac{1}{2}\rho_\infty U_\infty^{\ 2} C_l \, c \, dy \tag{2}$$

each chord strip will have a lift

$$C_{lx} = \left(1 - 2\frac{x}{c}\right)C_l \tag{3}$$

If we consider any strip dx (Fig. 3), the respective lift on an arc element ds will be

$$dL = \frac{1}{2}\rho_\infty U_\infty^{\ 2} C_{lx} \, dx \, ds \tag{4}$$

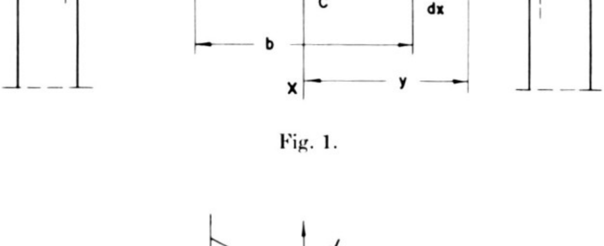

Fig. 1.

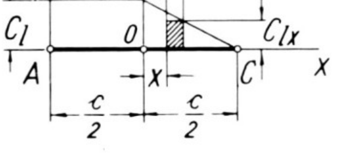

Fig. 2.

According to the momentum theorem this strip will camber and the radius R will be given by the relation

$$2\rho_j\delta_j U_j^{\,2} dx\frac{d\theta}{2} = \frac{1}{2}\rho_\infty U_\infty^{\,2} C_{lx}\, dx\, ds = \frac{1}{2}\rho_\infty U_\infty^{\,2} C_{lx} R\, dx\, d\theta \tag{5}$$

where ρ_j is the density, δ_j is the thickness and U_j is the fluid jet velocity; it is readily seen that the resulting radius is in inverse proportion with C_{lx},

$$R = 2\delta_j \frac{\rho_j}{\rho_\infty}\frac{U_j^{\,2}}{U_\infty^{\,2}}\frac{1}{C_{lx}} = \frac{R_0}{1 - 2\frac{x}{c}},\; R_0 = 2\delta_j \frac{\rho_j}{\rho_\infty}\frac{U_j^{\,2}}{U_\infty^{\,2}}\frac{1}{C_l} \tag{6}$$

where R_0 is the curvature radius of the middle of the plate.

The fluid plate warps; the leading edge and the middle of the plate follow each the radius given by Eq. (6), the trailing edge remaining the same.

Each strip dx lies on a circle tangent to oy at the point $y = b/2$, having as equation

$$(z - R)^2 + \left(y - \frac{b}{2}\right)^2 = R^2 \tag{7}$$

whence, at a point y of the span (Fig. 4) we may write approximately

$$z = R\left(1 - \sqrt{1 - \frac{\left(y - \frac{b}{2}\right)^2}{R^2}}\right) \approx \frac{\left(y - \frac{b}{2}\right)^2}{2R}\left(1 + \frac{1}{4}\frac{\left(y - \frac{b}{2}\right)^2}{R^2} + \cdots\right) \tag{8}$$

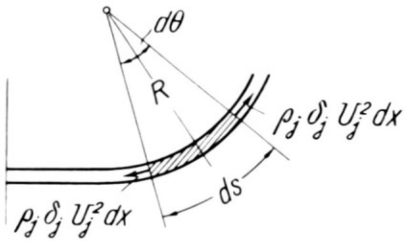

Fig. 3.

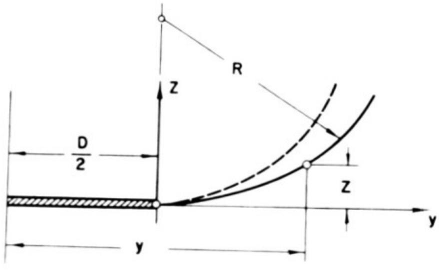

Fig. 4.

or further, if we restrict ourselves to the first term, and if R is replaced by its value given in Eq. (6), we find a chordwise linear variation

$$z = \left(1 - 2\frac{x}{c}\right)\frac{\left(y - \frac{b}{2}\right)^2}{2R_0} \tag{9}$$

It is thus seen that each section rotates by an angle ϵ; indeed, denoting by z_a the coordinate of the leading edge

$$z_a = \frac{\left(y - \frac{b}{2}\right)^2}{R_0} \tag{10}$$

the additional angle of incidence at point y, which adds to the incidence α_0 of the actual wing, will be

$$\epsilon = \alpha - \alpha_0 = \frac{\left(y - \frac{b}{2}\right)^2}{cR_0} \tag{11}$$

where α denotes the angle of incidence at a point y of the fluid wing.

It is seen thus that this incidence increases spanwise and the fluid wing cambers with the mean radius R_0, being meanwhile downstream directed. The phenomenon is very complex, but we have tried however to present it in a simple manner, in order to be able to establish at least a qualitative theoretical result.

In this sense, we shall also assume that the incidence increases up to a maximum value which will be denoted by α_m, and which corresponds to the maximum lift C_{lm}, beyond which, no matter what the variation of incidence would be, the lift decreases down to zero according to a very complicated law. Indeed, the fluid wing gets more and more warped and meanwhile cambers until finally the whole fluid jet is thrown towards downstream. The phenomena along the lateral fluid jet are very complex and very difficult to be established by a simple law. The principal aspects must be however retained, namely:

(*a*) The aspect ratio of the complex wing increases function of the jet, which results in the general increase of the circulation around the actual wing, this one constituting now the central part of a wing of high aspect ratio, and consequently the actual wing becomes more efficient;

(*b*) The warping of the jet flap increases the local incidence and produces in addition a new increase of circulation. On the other hand, we consider that the lateral jets also produce a boundary layer suction on the wing upper surface, which improves the flow.*

Using these findings, we shall try to construct a theoretical calculation scheme, by assimilating the actual wing and the plane fluid jet with a rectangular or elliptical wing, having the span B, which will be determined in an approximate manner.

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* In this respect, the boundary layer flow vizualization tests carried out on the wing upper surface at ONERA, by Werlé, Paris, may be cited.

It was shown above that the variation of lift, or otherwise expressed, the spanwise variation of circulation, is a very complex phenomenon; however in order to place it within Prandtl's theory of finite span wings, assumption will be made that the relation of incidence [Eq. (11)] is also valid beyond the incidence α_m, corresponding to the maximum lift C_{lm}. In such a case, if the actual wing has reached this incidence, the incidence on the fluid wing will vary according to relation [Eq. (11)] reaching at its new maximum value α_M at the wing end,

$$\alpha_M = \alpha_m + \frac{1}{cR_m}\left(\frac{B_m}{2} - \frac{b}{2}\right)^2 = \alpha_m + \frac{B_m^{\ 2}}{4cR_m}\left(1 - \frac{b}{B_m}\right)^2 \tag{12}$$

where B_m stands for the total span of the complex wing, which will be determined by using certain practical considerations.

Indeed, when the incidence of the actual wing reaches its maximum value α_m, the pressure is approximately chordwise uniformly distributed and the whole fluid plate cambers following a radius normal to its surface and equal to

$$R_m = 2\delta_j \frac{\rho_j}{\rho_\infty} \frac{U_j^{\ 2}}{U_\infty^{\ 2}} \frac{1}{C_{nm}} \tag{13}$$

where C_{nm} is the total coefficient of the aerodynamic resultant

$$C_{nm} = \sqrt{C_{lm}^{\ 2} + C_{dm}^{\ 2}} \approx C_{lm} \tag{14}$$

C_{dm} and C_{lm} being the drag coefficient (much smaller than C_{lm}) and the lift coefficient at the maximum lift angle α_m respectively. We shall use the simplifying assumption, which as a matter of fact, is very close to actuality, that for this position, half of the actual wing span is additionally prolonged, with the radius R_m.

We may thus write

$$B_m = b + 2R_m \tag{15}$$

whence α_M may be obtained. Simplifying further, the span B for any incidence α_0 of the actual wing may be determined by assuming that at the end of the complex wing the same incidence α_M will be obtained, given by Eq. (12); hence

$$\alpha_M = \alpha_m + \frac{B_m}{4cR_m}\left(1 - \frac{b}{B_m}\right)^2 = \alpha_0 + \frac{B}{4cR_0}\left(1 - \frac{b}{B}\right)^2 \tag{16}$$

This manner of setting the problem enables us to find at least a qualitative theoretical result which, in addition, takes into consideration the actual phenomenon.

Thus, under this simplifying scheme, the problem reduces to a rectangular wing or eventually to an elliptical wing, of span B, with the aspect ratio

$$\lambda = \frac{B}{c} \tag{17}$$

having the incidence α_0 on the actual wing portion, hence on the span b, and the incidence α given by Eq. (11) between $b/2$ and $B/2$, on each side of the complex wing span.

For solving this problem, we shall apply the usual method. We shall set first

$$y = -\frac{B}{2}\cos\theta \tag{18}$$

and the circulation around the wing will be developed into a Fourier series, function of θ,

$$\Gamma = 2BU_\infty \sum_1^n A_n \sin n\theta \tag{19}$$

We shall express then R_0 in Eqs. (6) and (16), function of the momentum coefficient, which will be denoted by C_μ, and defined as follows

$$C\mu = \frac{c\delta_j\rho_j U_j{}^2}{c\frac{b}{2}\frac{\rho_\infty}{2}U_\infty{}^2} = 4\frac{\delta_j}{b}\frac{\rho_j}{\rho_\infty}\left(\frac{U_j}{U_\infty}\right)^2 \tag{20}$$

where the subscript j refers to the jet. From Eq. (6) we deduce

$$R_0 = \frac{b}{2}\frac{C\mu}{C_{lb}} \tag{21}$$

where C_{lb} is the local lift at the end of the actual wing, just at the beginning of the jet, the circulation having the value Γ_b at the same point

$$C_{lb} = \frac{2\Gamma_b}{cU_\infty} = 4\frac{B}{c}\sum_1^n A_n \sin n\theta_b \tag{22}$$

and the angle θ_b representing the point $y = b/2$ on the span, is deduced from

$$\frac{b}{B} = |\cos\theta_b| \tag{23}$$

The radius R_m is given by the same formula, Eq. (21), where however C_{lb} is replaced by the maximum lift C_{lm},

$$R_m = \frac{b}{2}\frac{C\mu}{C_{lm}} \tag{24}$$

In this case, the incidence of the fluid wing becomes

$$\alpha = \alpha_0 + \frac{1}{2}\frac{C_{lb}}{C\mu}\frac{B^2}{b^2}\frac{b}{c}\left(|\cos\theta| - \frac{b}{B}\right)^2 \tag{25}$$

Taking also in consideration Eq. (23), we shall set

$$a_0 = \frac{1}{2}\frac{C_{lb}}{C\mu}\frac{B^2}{b^2}\frac{b}{c} = \frac{1}{2}\frac{C_{lb}}{C\mu}\frac{\lambda_0}{\cos^2\theta_b} \tag{26}$$

It is to be noticed however that the expression of a_0 has been deduced by considering a simple scheme of spanwise and chordwise lift distribution. Therefore, we shall affect a coefficient ν to the above expression and set

$$a = \nu a_0 = \frac{\nu}{2} \frac{C_{lb}}{C\mu} \frac{\lambda_0}{\cos^2 \theta_b} \tag{27}$$

the value of ν is to be estimated according to experimental indications.

In this case, the additional incidence $\epsilon = \alpha - \alpha_0$ given by Eq. (11) varies as follows

$$\epsilon = \alpha - \alpha_0 = 0 \qquad \text{from } 0 \text{ to } \frac{b}{2}$$

$$\epsilon = \alpha - \alpha_0 = a(\cos \theta - \cos \theta_b)^2 \text{ from } \frac{b}{2} \text{ to } \frac{B}{2}$$

From the development into Fourier series of the expression $\epsilon \sin \theta$,

$$\epsilon \sin \theta = \epsilon_1 \sin \theta + \epsilon_3 \sin 3\theta + \cdots + \epsilon_n \sin n\theta \tag{28}$$

we deduce

$$\epsilon_n = \frac{2a}{\pi} \left\{ \frac{b^2}{B^2} \left[\frac{\sin(n-1)\theta_b}{n-1} - \frac{\sin(n+1)\theta_b}{n+1} \right] - \frac{b}{B} \left[\frac{\sin(n-2)\theta_b}{n-2} - \frac{\sin(n+2)\theta_b}{n+2} \right] + \frac{1}{4} \left[\frac{\sin(n-1)\theta_b}{n-1} - \frac{\sin(n+1)\theta_b}{n+1} + \frac{\sin(n-3)\theta_b}{n-3} - \frac{\sin(n+3)\theta_b}{n+3} \right] \right\} \tag{29}$$

where n is an odd number ($n = 2p + 1$).

It is to be noticed that the problem reduces to a quasi-rectangular wing having all along the span the constant incidence α_0 to which the wing warping is added, which introduces an additional variable incidence ϵ on the entire portion from $b/2$ to $B/2$, on each side of the span. For the sake of simplification, this latter effect may be established as for an equivalent elliptical wing.

Further, in compliance with the usual notations, we shall set

$$\mu_0 = \frac{k}{2} \frac{c_0}{B} \qquad (k = 0.8\pi - 0.9\pi) \tag{30}$$

where k is half of the slope $dC_l/d\alpha$ for the wing of infinite span (theoretically $k \approx \pi$). However, the experimental slope is much smaller.

As regards the first effect, taking into consideration the fact that towards the jet end the geometrical form of the wing cannot be defined, but a rounded off shape may be however assumed, we shall consider a quasi-rectangular wing with rounded off ends, which may be quite well represented by the following relation

$$\frac{c}{c_o} \sin \theta = \beta_o - 2\beta_2 \cos 2\theta \tag{31}$$

where θ is defined by Eq. (18) and the coefficients β_0 and β_2 are respectively

$$\beta_o = 0.70 \qquad 2\beta_2 = 0.30 \tag{32}$$

Further, it is to be noticed that this equivalent wing has the incidence α_0 all along the span, as we have assumed from the beginning.

In this case, applying formulas previously obtained, we have calculated the coefficients A_1', A_3', . . . , A_N' corresponding to the constant incidence α_0 on the quasi-rectangular wing, and the additional coefficients A_1'', A_2'', . . . , A_N'' due to the warp of the fluid wing, calculated approximately as for an equivalent elliptical wing.

Taking into account the above considerations, we may set for each effect separately

$$\Gamma' = 2BU_\infty \sum_1^n A_n' \sin n\theta, \; \Gamma'' = 2BU_\infty \sum_1^n A_n'' \sin n\theta \tag{33}$$

and the total circulation Γ, defined by Eq. (19), is in this case

$$\Gamma = \Gamma' + \Gamma'' \tag{34}$$

Let us now proceed to the calculation of the lift of the actual wing of span b. The corresponding lift coefficient will be denoted by K_l and will be considered as consisting of two parts

$$K_l = K_l' + K_l'' \tag{35}$$

where K_l' is the mean lift coefficient of the wing of span b, assumed to be the middle part of a rectangular wing of span $B > b$ and of constant incidence α_0, while K_l'' is the lift coefficient of the same wing of span b, due to the jet warping.

The calculation of K_l' and K_l'' is very simple and results from the following general formulas

$$\begin{aligned} \frac{1}{2}\rho_\infty U_\infty^2 \frac{b}{2} cK_l' &= \rho_\infty U_\infty \int_0^{b/2} \Gamma' dy \\ \frac{1}{2}\rho_\infty U_\infty^2 \frac{b}{2} cK_l'' &= \rho_\infty U_\infty \int_0^{b/2} \Gamma'' dy \end{aligned} \tag{36}$$

As it is seen, the coefficients K_l' and K_l'' are functions of B or of the ratio $b/B = |\cos\theta_b|$ and of C_μ. For carrying out the calculation, we must determine C_{lb} at the end of the actual wing [Eq. (22)], in order to be able then to find the value of a [Eq. (27)].

The final expressions of K_l are obtained by using the circulation coefficients

$$\begin{aligned} K_l &= \frac{2B^2}{bc}\sum_1^n (A_n' + A_n'') \int_{\theta_b}^{\pi/2} [\cos(n-1)\theta - \cos(n+1)\theta] d\theta \\ &= \frac{2B^2}{bc}\Bigg[(A_1' + A_1'')\left(\frac{\pi}{2} - \theta_b + \frac{\sin 2\theta_b}{2}\right) \\ &\quad + \sum_{n\geq 3} (A_n' + A_n'')\left(\frac{\sin(n+1)\theta_b}{n+1} - \frac{\sin(n-1)\theta_b}{n-1}\right)\Bigg] \end{aligned} \tag{37}$$

Since in calculations θ_b and $C\mu$ have been considered independent variables, it is necessary to find a relation between these variables and the incidence α_0.

In this sense, we have assumed that at the wing end, that is at point $y = B/2$, the incidence assumes the same value for each jet.

The diagrams showing the lift coefficient against the incidence variation have been plotted for various values of the jet coefficient $C\mu$.

The calculations are not difficult, but are laborious. Performing these calculations under certain conditions, the diagrams shown in Fig. 5 have been plotted, which agree at least qualitatively with the experimental results obtained at the Aerodynamics Laboratory of the Institute of Applied Mechanics in Bucharest. In view of the complexity of the jet-main stream interaction phenomenon which has imposed the conceiving of a simple scheme for treating theoretically this problem, the agreement may be considered as fairly good.

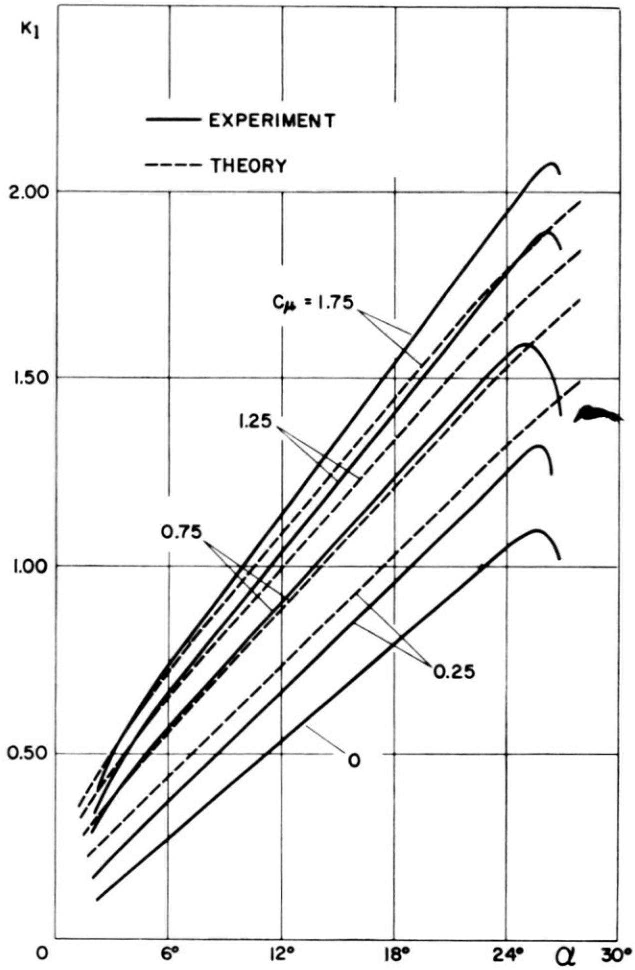

Fig. 5. Lift against incidence for lateral jet ($\gamma = 0$).

Figure 6 shows the variation of the same pure lift coefficient (which does not include the vertical jet reaction) against various values of the jet coefficient, a $\gamma = 45°$ jet deflection with respect to the horizontal being however imposed. It is worth noting that in comparison with the horizontal jet, the deflected jet presents the advantage of increasing the wing lift by its reaction, and is recovered in proportion of nearly $\cos 45° = 0.71$. Indeed, the lift resulting from the jet aerodynamic effect remains approximately as for the horizontal jet; consequently, it is recommendable to use lateral jets deflected downwards with larger angles, even larger than $\gamma = 45°$. Of course, new investigations on the behavior of the plane fluid wing will result in better possibilities of situating the phenomenon and applying the theory of finite span wings by using a scheme more close to the actual phenomenon.

THE CASE OF VERY LOW ASPECT RATIOS

Even if the aspect ratio of the actual wing is very low, the virtual aspect ratio increases substantially through the jet effect and thus the theory of finite span wings may be applied for calculating, as above, its aerodynamic characteristics. In such a case, however, the following simplifying approximations can be made:

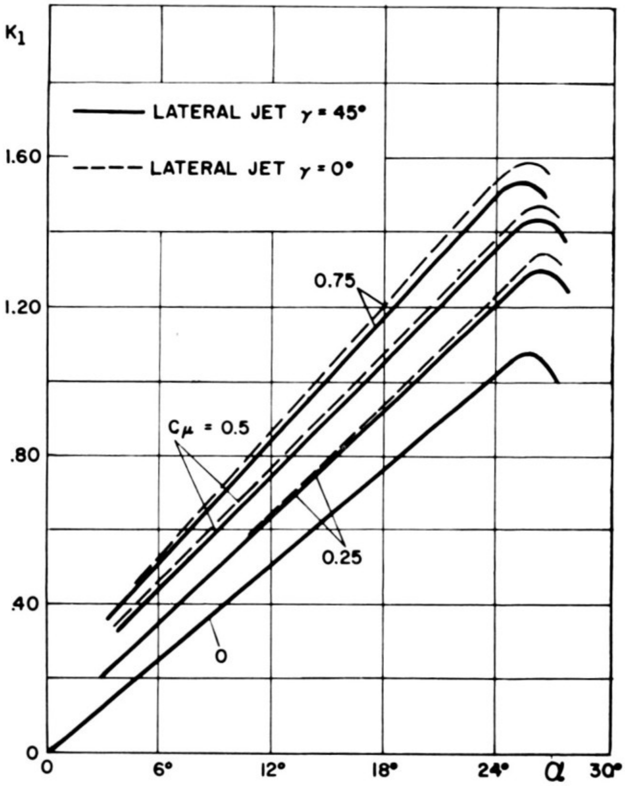

Fig. 6. Lift coefficient due to the deflected lateral jet.

(*a*) The complex wing with lateral fluid jet may be considered as having an elliptical shape, with the median chord equal to that of the actual wing.

(*b*) The mean lift coefficient of the actual wing, which will be again denoted by K_l, may be taken approximately equal to that of the middle section of the wing, that is C_{l0}.

(*c*) The lift coefficient at the wing end, denoted above by C_{lb}, may be also replaced by C_{l0} (hence by K_l), since C_{l0} and C_{lb} do not differ sensibly, due to the fact that the wing span being small, the variation of C_l is small between the middle and the end of the wing.

(*d*) The complex wing total span may increase on both sides, by at most R_0, i.e., the jet curvature radius.

Denoting hence by κ a coefficient smaller than unity, we may write for the total span B,

$$B = b + 2\kappa R_0 \tag{38}$$

As on the other hand, replacing C_{lb} by K_l in Eq. (21), we may write

$$R_0 = \frac{b}{2}\frac{C\mu}{K_l} \tag{39}$$

we obtain finally

$$B = b + \kappa b \frac{C\mu}{K_l} = b\left(1 + \kappa \frac{C\mu}{K_l}\right) \tag{40}$$

If an elliptical wing is assumed, the circulation coefficients, Γ' and Γ'', will be respectively

$$A_1' = \frac{\mu_0\alpha_0}{1 + \mu_0} \qquad A_n' = 0 \qquad n > 1 \tag{41a}$$

$$A_n'' = \frac{\mu_0\epsilon_n}{1 + n\mu_0} = \frac{a\mu_0\alpha_n}{1 + n\mu_0} \tag{41b}$$

where ϵ_n is given by Eq. (29), α_n results from Eq. (29), and μ_0 is given by Eq. (30), c_0 being the actual wing chord, which will be denoted currently by c without subscript.

According to our assumption, we may write as for Eq. (22), where however $\theta = \pi/2$, instead of θ_b,

$$K_l = C_{l_0} = \frac{2\Gamma_0}{cU_\infty} = 4\frac{B}{c}(A_1 - A_3 + A_5 - \cdots)$$

$$= 4\frac{B}{c}\frac{\mu_0\alpha_0}{1 + \mu_0} + 4\frac{B}{c}a\mu_0\sum_1^n \frac{(-1)^{(n-1)/2}\alpha_n}{1 + n\mu_0} \tag{42}$$

or further,

$$K_l = 2k\left[\frac{\alpha_0}{1 + \mu_0} + a\sum_1^n \frac{(-1)^{(n-1)/2}\alpha_n}{1 + n\mu_0}\right] \tag{43}$$

If we observe now that, taking into consideration Eq. (30), we have successively

$$\mu_0 = \frac{k}{2}\frac{c}{B} = \frac{k}{2}\frac{c}{b}\frac{b}{B} = \frac{k}{2\lambda_0\left(1 + \kappa\dfrac{C_\mu}{K_l}\right)} \tag{44}$$

and then, taking into consideration Eq. (26),

$$a = \frac{\lambda_0}{2}\frac{K_l}{C\mu}\frac{B^2}{b^2} = \frac{\lambda_0}{2}\frac{K_l}{C\mu}\left(1 + \kappa\frac{C_\mu}{K_l}\right)^2, \ (\gamma = 1) \tag{45}$$

noting on the other hand that in the expression of ϵ_n (or of α_n) the angle θ_b appears, deduced from Eq. (23),

$$\theta_b = \text{arc cos } b/B = \text{arc cos}\left(\frac{1}{1 + \kappa\dfrac{C_\mu}{\kappa_l}}\right) \tag{46}$$

expression (43) may be finally written

$$2k\alpha_0 = K_l\left[1 + \frac{k}{2\lambda_0\left(1 + \kappa\dfrac{C_\mu}{K_l}\right)}\right] \left[1 - \frac{\lambda_0 k}{C_\mu}\left(1 + \kappa\frac{C_\mu}{K_l}\right)^2 \sum_1^n \frac{(-1)^{(n-1)/2}\alpha_n}{1 + \dfrac{nk}{2\lambda_0\left(1 + \kappa\dfrac{C_\mu}{K_l}\right)}}\right] \tag{47}$$

Under this form, the calculation of K_l function of α_0 may be easier performed.

Indeed, from the same coefficient $C\mu$, one may proceed from small values of K_l ($K_1 > 0, 10$), which generally are compatible to Eq. (47) and we calculate first the ratio $C\mu/K_l$, and then $\cos\theta_b = b/B$ or the angle θ_b. Introducing these values into the above expressions, we obtain α_0.

COMPARISON BETWEEN THE EFFECTS OF THE LATERAL AND LONGITUDINAL JETS UPON THE LIFT

In order to emphasize the effect of the lateral and longitudinal jets upon the pressure on the wing and hence upon the pressure lift, that is without considering the effect of the vertical momentum, the diagrams showing the variation of lift against the angle of incidence have been plotted for various values of the coefficient $C\mu$ (Fig. 7). It is to be noticed that although the aspect ratio $\lambda = 2$ of the actual wing is relatively moderate, one may see that the effect of the lateral jet is of importance.

Figure 7 shows a comparison between the lateral jet and the longitudinal one, the jet deviation with respect to the profile axis being $\beta = 0$. Of course, for large values of β, the action of the longitudinal jet becomes of importance; nevertheless, as it will be seen below, for combined jets, the effect of the lateral jet is very high.

RECTANGULAR WING WITH COMBINED FLUID JETS

By a series of tests carried out on the wing of aspect ratio $\lambda_0 = 2$, we have illustrated the effect of the combined lateral and longitudinal jets.[7] In order that the jet sheet be not discontinuous, we have chosen a fanning out distribution (Fig. 8), the angular jet intensity being approximately the same. It is rather difficult to situate this jet distribution within a theory, but in the light of the above theoretical considerations, an application of the obtained results can be made, by introducing similar assumptions as regards the behavior of the jet sheet both in lateral and longitudinal direction. In this sense, as in the case of lateral jets, a similar formula may be found for the lift of the rectangular wing with combined jets.

The experimental results obtained for combined jets are plotted in the diagrams shown in Fig. 9 ($\gamma = \beta = 0$) and Fig. 10 ($\gamma = \beta = 45°$).

An examination of the diagrams shows readily that the wing lift due to the combined jets exceeds that due to the simple longitudinal jets; the additional lift as compared to the jetless wing is greater for the wings with combined jets than for that with simple longitudinal jets.

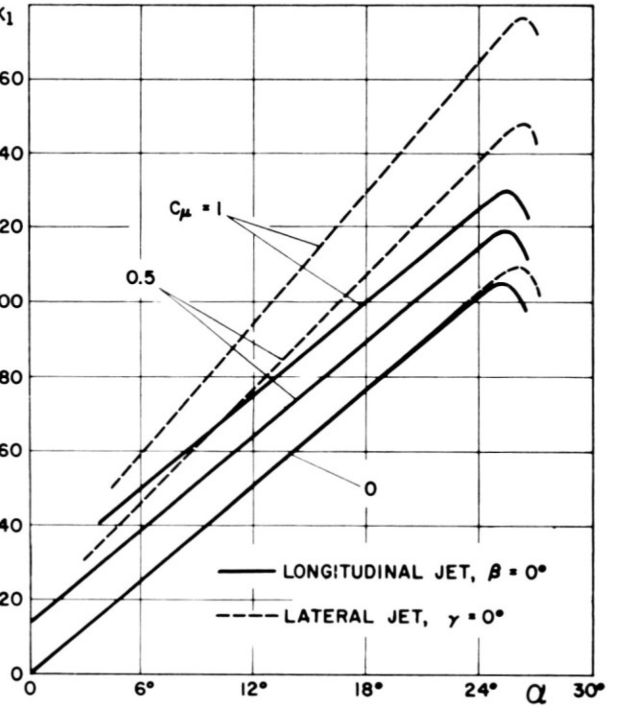

Fig. 7. Comparison of lifts due to the longitudinal and lateral jets.

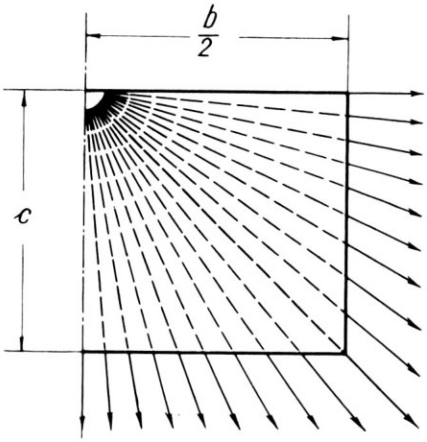

Fig. 8.

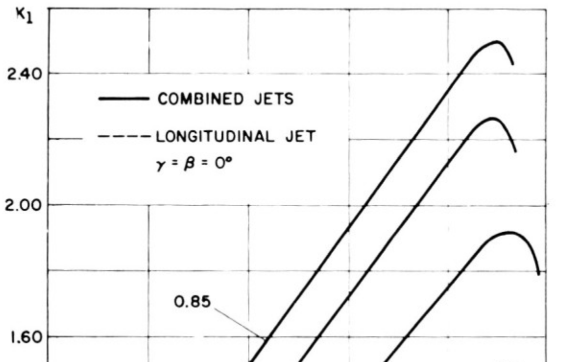

Fig. 9. Comparison of lifts due to combined and longitudinal jets.

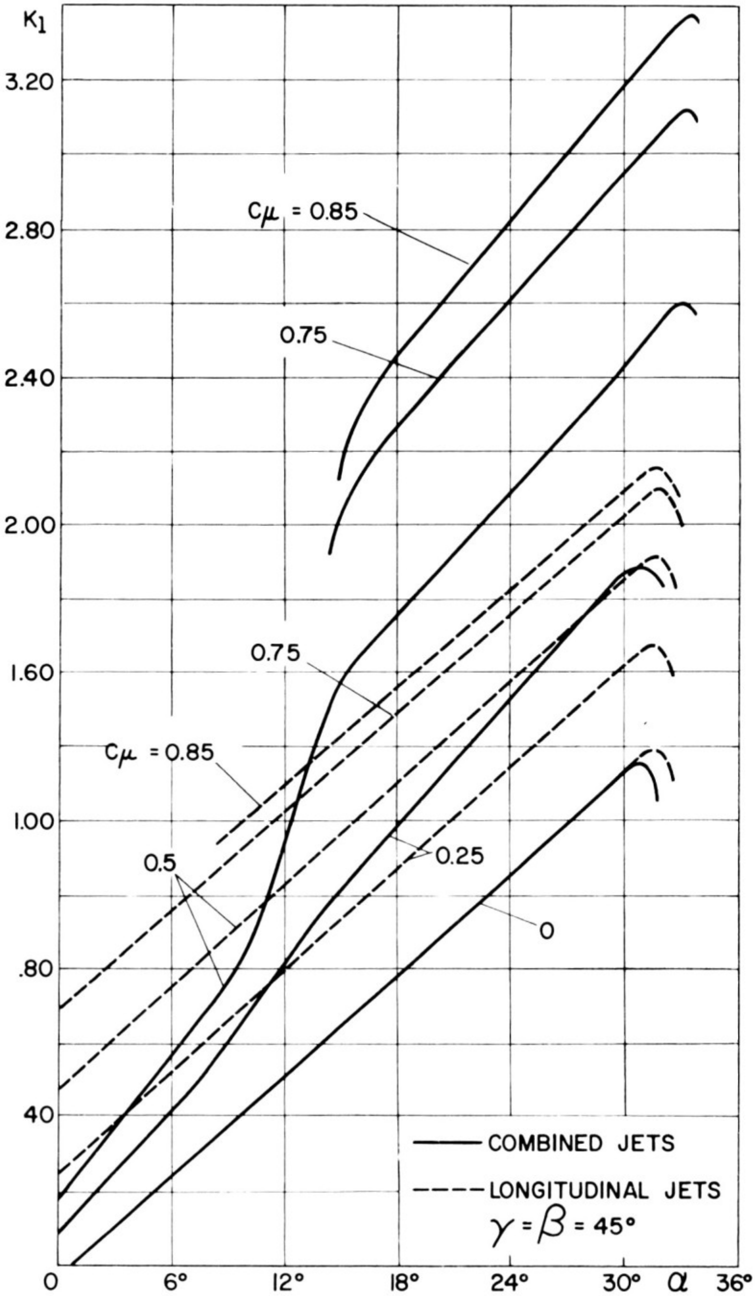

Fig. 10. Lift against incidence for combined jets ($\gamma = \quad = 45°$) and for simple longitudinal jet ($\gamma = 45°$).

CONCLUSIONS

The above considerations aimed at emphasizing the effect of lateral jets, simple or combined with longitudinal jets. For technical reasons, the tests have been carried out on wings of moderate aspect ratio ($\lambda_0 = 2$); but as noticed above, the lateral jets have, proportionally, a greater effect in the case of wings of low aspect ratio. Therefore, for slender arrow-shaped wings, which present some difficulties at landing due to the small lateral control, the lateral jet constitutes a requirement in order to overcome this disadvantage.

The application of the lateral jets may be extended even to bodies of revolution (fuselages or rockets without wings or fitted with very small wings), which, by the very effect of the lateral jets eventually combined with a longitudinal jet system, may become lifting bodies, increasing thus the effect of the jet vertical component in case of short landing.

In this respect, our investigations have been extended to various cases which we thought useful for practical applications; they will constitute the object of subsequent papers.

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DISCUSSION

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Félicite les auteurs d'avoir entrepris l'étude théorique des jets latéraux, mais souligne l'importance des effets non linéaires, au moins pour les jets perpendiculaires à l'aile spécialement étudiées en France. La portance fournie par un soufflage au bord de fuite combiné avec un soufflage latéral est supérieure à la somme des portances pour ces soufflages employés isolément.

La disposition s'adapte mal aux ailes delta qui peuvent cependant être prolongées par des élévoirs en demi ellipse avec soufflage sur tout le bord de fuite. Les difficultés d'équilibrage longitudinal sont notables.

Un effet secondaire important du soufflage latéral est le balayage des décollement d'extremités d'aile qui assure une meilleure continuité d'évolution du moment et améliore la stabilité longitudinale.

Author's reply to discussion:

At present, we are carrying out different tests concerning lateral and combined jets on various planforms: delta wing, circular wing, rectangular wing, etc., with interesting results, which will make the subject of subsequent papers.